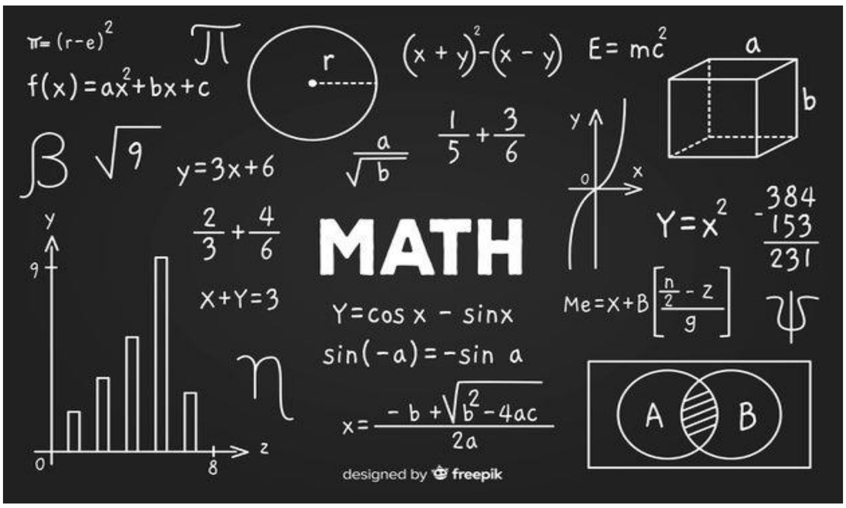
Higham Lane School

A Level Maths

Transitional Skills Booklet



Reading List

As a student who is choosing to study A Level Maths, it’s logical to assume you have an interest in the subject. The following books may provide additional reading for you.

Alex’s Adventures in Numberland *by Alex Bellos*

Cabinet of Mathematical Curiosity *by Ian Stewart*

The num8er My5teries *by Marcus De Sautoy*

How Many Socks Make a Pair? Surprisingly Interesting Maths *by Rob Eastaway*

The Curious Incident of The Dog in the Night-time *by Mark Haddon*

The Penguin Dictionary of Curious and Interesting Numbers *by David Wells*

The Calculus Wars *by Jason Socrates Bardi*

The Code Book *by Simon Singh*

50 Mathematical Ideas You Should Really Know *by Tony Crilly*

Skills Check List

- Indices
- Surds
- Factoring
- Rearranging formula
- Completing the Square
- Solving Quadratic Equations
- Solving Linear Equations
- Simultaneous Equations
- Straight Line Graphs
- Quadratic Graphs
- Other Graphs
- Inequalities
- Trigonometry (including Pythagoras)

INDICES	
What you should know from GSCE	**Video links to help**
•To write an exponent on a calculator •To understand zero and negative indices •To apply the laws of indices for multiplying and dividing, and for powers of indices •To work with fractional indices and understand the link to surds •To calculate roots of a number •To solve problems involving powers and roots	https://app.mymaths.co.uk/153-lesson/indices-1 https://app.mymaths.co.uk/1785-lesson/indices-2 https://app.mymaths.co.uk/154-lesson/indices-3 https://app.mymaths.co.uk/155-lesson/indices-4 https://corbettmaths.com/2013/03/13/laws-of-indices-algebra/ https://corbettmaths.com/2013/03/24/negative-indices/

Laws of indices

$$a^m \times a^n = a^{m+n} \qquad a^0 = 1 \qquad a^{\frac{1}{2}} = \sqrt{a}$$

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n} \qquad a^{-1} = \frac{1}{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

$$\left(a^m\right)^n = a^{mn}$$

EXAMPLES

Simplifying

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the a's)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the c's)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the d terms by subtracting the powers)

Negative powers

$$5^{-1} = \frac{1}{5}$$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$$

(Find the reciprocal of a fraction by turning it upside down)

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad 2^{-4} = \frac{1}{2^4} = \frac{1}{16} \qquad \left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers

$8^{1/3} = \sqrt[3]{8} = 2$ $\qquad 25^{1/2} = \sqrt{25} = 5$ $\qquad 10000^{1/4} = \sqrt[4]{10000} = 10$

$$4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Exercise A - simplifying

1) $b \times 5b^5$

2) $3c^2 \times 2c^5$

3) $b^2c \times bc^3$

4) $2n^6 \times (-6n^2)$

5) $8n^8 \div 2n^3$

6) $d^{11} \div d^9$

7) $\left(a^3\right)^2$

8) $\left(-d^4\right)^3$

Exercise B - Evaluate

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(1/9\right)^{1/2}$

4) 5^{-2}

5) 18^0

6) 7^{-1}

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify

13) $2a^{1/2} \times 3a^{5/2}$

14) $x^3 \times x^{-2}$

15) $\left(x^2y^4\right)^{1/2}$

Problem Solving

These are non-calculator exercises.

1) $4^6 \times 8^7 = 2^x$ Find x

2) $\frac{2^7}{\sqrt{2}} = 2^x$ Find x

3) $\frac{\sqrt[3]{9}}{81^2} = 9^x$ Find x

4) $\sqrt{128} \times 8^3 = \frac{16^{x-8}}{4}$ Find x

5) $(x^6)^m = \frac{(x^m)^m}{x^7}$ Find m

6) $\sqrt{\frac{1}{x^4}} = \frac{x^4 \times x^2}{(x^a)^4}$ Find a

SURDS	
What you should know from GSCE	**Video links to help**
•To use a calculator to approximate the values of numbers involving surds •To calculate exact solutions to problems using surds •To simplify expressions containing surds •To manipulate surds when multiplying and dividing •To rationalise the denominator of a fraction •To apply an understanding of surds to solve more complex problems	https://app.mymaths.co.uk/156-lesson/surds-1 https://app.mymaths.co.uk/157-lesson/surds-2 https://corbettmaths.com/2013/05/11/surds/ https://corbettmaths.com/2013/05/11/surds-addition/ https://corbettmaths.com/2013/05/11/rationalising-denominators/ https://corbettmaths.com/2013/05/11/surds-expanding-brackets/

Laws of Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

EXAMPLES

Simplifying

$$\begin{aligned}\sqrt{8} &= \sqrt{4} \times \sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

$$\begin{aligned}3\sqrt{12} &= 3 \times \sqrt{4} \times \sqrt{3}\\ &= 3 \times 2 \times \sqrt{3}\\ &= 6\sqrt{3}\end{aligned}$$

$$\begin{aligned}\frac{\sqrt{600}}{\sqrt{2}} &= \sqrt{\frac{600}{2}}\\ &= \sqrt{300}\\ &= \sqrt{100} \times \sqrt{3}\\ &= 10\sqrt{3}\end{aligned}$$

Multiplying and dividing

$$2\sqrt{3} \times \sqrt{2} = 2\sqrt{6}$$

$$3\sqrt{5} \times 6\sqrt{2} = 18\sqrt{10}$$

$$\begin{aligned}2\sqrt{5} \times 7\sqrt{8} &= 14\sqrt{40}\\ &= 14 \times \sqrt{4} \times \sqrt{10}\\ &= 28\sqrt{10}\end{aligned}$$

$$\sqrt{2}(5 + 2\sqrt{3}) = 5\sqrt{2} + 2\sqrt{6}$$

$$\frac{8\sqrt{14}}{2\sqrt{7}} = 4\sqrt{2}$$

$$(1 + \sqrt{3})(2 - \sqrt{2}) = 2 - 2\sqrt{2} + 2\sqrt{3} - \sqrt{6}$$

$$\begin{aligned}(3 + \sqrt{2})(3 - \sqrt{2}) &= 3^2 - (\sqrt{2})^2\\ &= 1\end{aligned}$$

Adding and Subtracting

$$2\sqrt{3}+4\sqrt{3}+6\sqrt{5}=6\sqrt{3}+6\sqrt{5}$$

Here add the $2\sqrt{3}$ and $4\sqrt{3}$ as the same surd is present but you cannot add the $6\sqrt{5}$.

$$\begin{aligned}2\sqrt{5}+\sqrt{45}&=2\sqrt{5}+3\sqrt{5}\\&=5\sqrt{5}\end{aligned}$$

By simplifying $\sqrt{45}$ to $3\sqrt{5}$, you can add the two surds together.

Rationalising

$\frac{1}{\sqrt{3}}$ Multiply the denominator by $\sqrt{3}$ to rationalise it and so multiply the numerator by $\sqrt{3}$ also:

$$\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\begin{aligned}\frac{4}{\sqrt{2}}&=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{4\sqrt{2}}{2}\\&=2\sqrt{2}\end{aligned}$$

$$\begin{aligned}\frac{2+\sqrt{3}}{\sqrt{5}}&=\frac{2+\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}\\&=\frac{\sqrt{5}(2+\sqrt{3})}{5}\\&=\frac{2\sqrt{5}+\sqrt{15}}{5}\end{aligned}$$

Rationalise $\frac{2}{3-\sqrt{7}}$

We multiply the numerator and denominator by its conjugate: $3+\sqrt{7}$

It's a difference of two squares so expand as usual

$$\begin{aligned}\frac{2}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}&=\frac{2(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}\\&=\frac{2(3+\sqrt{7})}{3^2-(\sqrt{7})^2}\\&=\frac{2(3+\sqrt{7})}{9-7}\\&=\frac{2(3+\sqrt{7})}{2}\\&=3+\sqrt{7}\end{aligned}$$

Exercise A – simplify

1) $\sqrt{50}$
2) $\sqrt{72}$
3) $\sqrt{27}$
4) $\sqrt{80}$
5) $\sqrt{360}$
6) $\frac{\sqrt{900}}{\sqrt{3}}$

Exercise B – multiplying and dividing

1) $\sqrt{3}\times\sqrt{7}$
2) $5\sqrt{2}\times4\sqrt{5}$
3) $3\sqrt{3}\times2\sqrt{6}$
4) $\sqrt{8}\times\sqrt{27}$
5) $\frac{5\sqrt{20}}{6\sqrt{5}}$
6) $\frac{8\sqrt{18}}{4\sqrt{2}}$
7) $(\sqrt{2}+1)(\sqrt{2}+5)$
8) $(5-\sqrt{3})(\sqrt{2}-8)$

Exercise C – Adding and subtracting

1) $\sqrt{3}+\sqrt{7}$
2) $5\sqrt{2}+4\sqrt{2}$
3) $3\sqrt{6}+\sqrt{24}$
4) $\sqrt{50}+\sqrt{8}$
5) $\sqrt{27}+\sqrt{75}$
6) $2\sqrt{5}-\sqrt{5}$
7) $\sqrt{72}-\sqrt{50}$
8) $6\sqrt{3}-\sqrt{12}+\sqrt{27}$
9) $\sqrt{200}+\sqrt{90}-\sqrt{98}$
10) $\sqrt{72}-\sqrt{75}+\sqrt{108}$

Exercise D – Rationalising

Rationalise the following:

1

a) $\frac{1}{\sqrt{2}}$ b) $\frac{3}{\sqrt{5}}$ c) $\frac{10}{\sqrt{5}}$

d) $\frac{5}{2\sqrt{7}}$ e) $\frac{\sqrt{3}}{\sqrt{2}}$ f) $\frac{10}{\sqrt{10}}$

g) $\frac{4+\sqrt{7}}{\sqrt{3}}$ h) $\frac{6+8\sqrt{5}}{\sqrt{2}}$ i) $\frac{6-\sqrt{5}}{\sqrt{5}}$

2

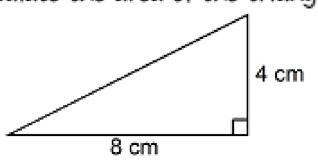
a) $\frac{1}{\sqrt{2}-1}$ b) $\frac{2}{\sqrt{6}-2}$ c) $\frac{6}{\sqrt{7}+2}$

d) $\frac{1}{3+\sqrt{5}}$ e) $\frac{1}{\sqrt{6}-\sqrt{5}}$

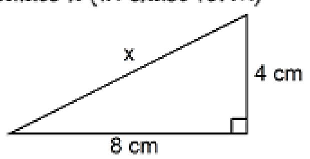
Problem Solving

Task 1 –structured

Calculate the area of the triangle

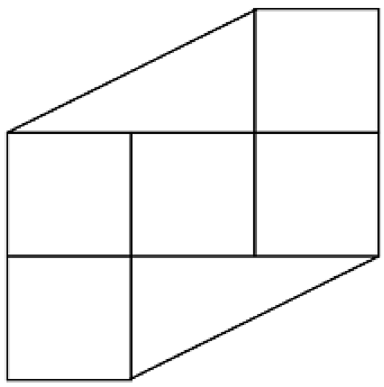


Calculate x (in exact form)



Simplify

$4\sqrt{3}+5+4\sqrt{3}+10$



The design shown, consisting of 5 squares and 2 triangles, has an area of 175 cm²

Calculate in exact form the perimeter of the design

Task 2

A rectangle has area $14cm^2$. Two of its sides each measure $(4 + \sqrt{2})$cm.

a) Find the length of the other sides of the rectangle.
Give your answer in the form $a + b\sqrt{c}$ where a, b and c are integers.

b) Hence, find the length of the diagonal of the rectangle.

Task 3

a) This is a geometric sequence:
$2, 2\sqrt{3}, 6, 6\sqrt{3}, \ldots$

i. Write down the common ratio.

ii. Calculate the next three terms of the sequence.

b) The n^{th} term of a geometric sequence is given by ar^{n-1}, where a is the first term and r is the common ratio. Find the n^{th} term of the following sequence, giving your answer in its simplest form.

$1, \sqrt{2}, 2, 2\sqrt{2}, \ldots$

FACTORING	
What you should know from GSCE	**Video links to help**
•To factorise expressions by taking out common factors and recognise that the HCF must be used for an expression to be fully factorised •To be able to factorise expressions of the form $x^2 + bx + c$ •To be able to factorise expressions of the form $ax^2 + bx + c$ •To form algebraic expressions to solve problems	https://app.mymaths.co.uk/173-lesson/factorising-linear https://app.mymaths.co.uk/174-lesson/factorising-quadratics-1 https://app.mymaths.co.uk/175-lesson/factorising-quadratics-2 https://corbettmaths.com/2013/02/06/factorisation/ https://corbettmaths.com/2013/02/06/factorising-quadratics-1/ https://corbettmaths.com/2013/02/07/factorising-quadratics-2/ https://corbettmaths.com/2019/03/26/splitting-the-middle-term/ https://corbettmaths.com/2013/02/08/difference-between-two-squares/

Examples

Example 1: Factorise $6x^2 - 2xy$

Solution: 2 is a common factor to both 6 and 2.
Both terms also contain an x.
Factorise by taking $2x$ outside a bracket
$$6x^2 - 2xy = 2x(3x - y)$$

Example 2: Factorise $9x^3y^2 - 18x^2y$

Solution: 9 is a common factor to both 9 and 18.
The highest power of x that is present in both expressions is x^2.
There is also a y present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:
$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

Example 3: Factorise $3x(2x - 1) - 4(2x - 1)$

Solution: There is a common bracket as a factor.
So we factorise by taking $(2x - 1)$ out as a factor.
The expression factorises to $(2x - 1)(3x - 4)$

Example 4: Factorise $x^2 - 9x - 10$.

Solution: Find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
One method is that of combining factors. Look at factorising on MyMaths or ask a teacher for help with our preferred method but is difficult to explain on paper.
Another method is:

Step 1: Find two numbers that multiply together to make ac and add to make b.
Step 2: Split up the bx term using the numbers found in step 1.
Step 3: Factorise the front and back pair of expressions as fully as possible.
Step 4: There should be a common bracket. Take this out as a common factor.

Example : Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x}\ \underbrace{+\,9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + (a)(x - (a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$

$16x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x+4)(x-4)$

and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x+4y)(x-4y)$

Exercise A - Factorise

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y-1) + 3(y-1)$

Exercise B - Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

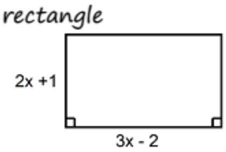
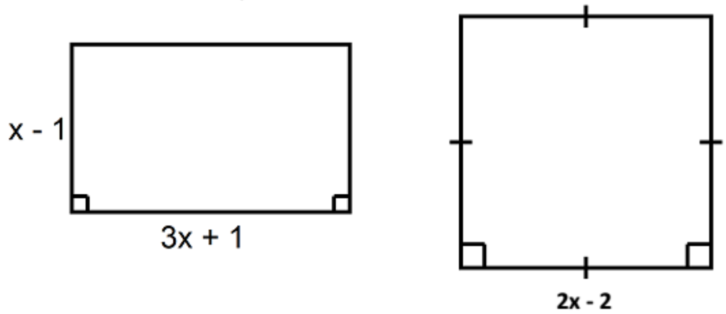
13) $4y^3 - 9a^2y$

14) $8(x+1)^2 - 2(x+1) - 10$

Problem Solving –Task 1

The area of a rectangle is given by the expression $6x^2 + 3x$. Find an expression for the width and length of the rectangle.	The area of a rectangle is given by the expression $x^2 + 8x - 20$. Find an expression for the width and length of the rectangle. Hence write down a condition for the value of x.

Task 2

<table>
<tr>
<td>Factorise
$x^2 - 7x + 12$</td>
<td>Write an expression for the area of the rectangle

</td>
<td>Expand and simplify $(3x + 2)^2$</td>
</tr>
<tr>
<td colspan="3">The square has the same area as the rectangle.
Calculate the perimeter of the square.

</td>
</tr>
</table>

REARRANGING FORMULA	
What you should know from GSCE	**Video links to help**
•To be able to rearrange formulae to change the subject up to where formulae include algebraic fractions (all learners need simple ones where the subject appears more than once) **•To rearrange formulae to change the subject where the formula includes algebraic fractions**	https://app.mymaths.co.uk/206-lesson/rearranging-1 https://app.mymaths.co.uk/207-lesson/rearranging-2 https://corbettmaths.com/2013/12/23/changing-the-subject-video-7/ https://corbettmaths.com/2013/12/28/changing-the-subject-advanced-video-8/

Examples

Example 1: Make x the subject of the formula $y = 4x + 3$.

Solution: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with x the subject.

Example 2: Make x the subject of $y = 2 - 5x$

Solution: Notice that in this formula the x term is negative.

$y = 2 - 5x$

Add $5x$ to both sides $y + 5x = 2$ (the x term is now positive)

Subtract y from both sides $5x = 2 - y$

Divide both sides by 5 $x = \frac{2-y}{5}$

Example 3: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

Rearrange to make F the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9 $9C = 5(F - 32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C+160}{5}$.

Harder Examples

Example 4: Make x the subject of $x^2 + y^2 = w^2$

Solution: $x^2 + y^2 = w^2$

Subtract y^2 from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving x)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember the positive & negative square root.

Example 5: Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\frac{5a}{h}}$

Square both sides $16t^2 = \frac{5a}{h}$

Multiply by h: $16t^2h = 5a$

Divide by 5: $\frac{16t^2h}{5} = a$

Sometimes the subject occurs in more than one place in the formula. In these questions collect the terms involving this variable on one side of the equation, and put the other terms on the opposite sid

Example 6: Make t the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add xt to both sides: $a = b + yt + xt$

Now put the terms without a t on the left hand side:

Subtract b from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

Example 7: Make W the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. Begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \frac{2bT}{a + 2b}$

Exercise A – Make *x* the subject of these formula

1) $y = 7x - 1$

2) $y = \frac{x+5}{4}$

3) $4y = \frac{x}{3} - 2$

4) $y = \frac{4(3x-5)}{9}$

Exercise B – Make t the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

Exercise C – Make *x* the subject of each of the following

1) $ax + 3 = bx + c$

2) $3(x+a) = k(x-2)$

3) $y = \frac{2x+3}{5x-2}$

4) $\frac{x}{a} = 1 + \frac{x}{b}$

Challenge

1 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

2 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

3 Make x the subject of the following equations.

a $\frac{p}{q}(sx+t) = x - 1$

b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

COMPLETING THE SQUARE	
What you should know from GCSE	**Video links to help**
•To complete the square on a quadratic expression •To identify the turning point (vertex) of a quadratic	https://app.mymaths.co.uk/193-lesson/completing-the-square https://corbettmaths.com/2013/12/29/completing-the-square-video-10/ https://corbettmaths.com/2017/09/25/quadratic-graphs-completing-the-square/

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$

$= (x + 3)^2 - 9 - 2$

$= (x + 3)^2 - 11$

1 Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

2 Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$

$= 2\left(x^2 - \frac{5}{2}x\right) + 1$

$= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$

$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$

$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$

1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$

2 Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2

4 Simplify

If the coefficient of x^2 is a perfect square you can sometimes get a more useful form.

Example 3 Write $4x^2 + 20x + 19$ in the form $(ax + b)^2 + c$.

Solution It should be obvious that $a = 2$ (the coefficient of a^2 is 4).

So $4x^2 + 20x + 19 = (2x + b)^2 + c$

If you multiply out the bracket now, the middle term will be $2 \times 2x \times b = 4bx$.

So $4bx$ must equal $20x$ and clearly $b = 5$.

And we know that $(2x + 5)^2 = 4x^2 + 20x + 25$.

So
$$\begin{aligned} 4x^2 + 20x + 19 &= (2x + 5)^2 - 25 + 19 \\ &= (2x + 5)^2 - 6. \end{aligned}$$

Exercise A

1 Write the following in the form $(x + a)^2 + b$.

(a) $x^2 + 8x + 19$ (b) $x^2 - 10x + 23$ (c) $x^2 + 2x - 4$

(d) $x^2 - 4x - 3$ (e) $x^2 - 3x + 2$ (f) $x^2 - 5x - 6$

2 Write the following in the form $a(x + b)^2 + c$.

(a) $3x^2 + 6x + 7$ (b) $5x^2 - 20x + 17$ (c) $2x^2 + 10x + 13$

3 Write the following in the form $(ax + b)^2 + c$.

(a) $4x^2 + 12x + 14$ (b) $9x^2 - 12x - 1$ (c) $16x^2 + 40x + 22$

Finding Turning Points

When $y = (x + a)^2 + b$ then the coordinates of the turning point is $(-a, b)$.
The minimum or maximum value of y is b.

Example 1:

Given $y = x^2 + 6x - 5$, by writing it in the form $y = (x + a)^2 + b$, where a and b are integers, write down the coordinates of the turning point of the curve. Hence sketch the curve.

$y = x^2 + 6x - 5$

$= (x + 3)^2 - 9 - 5$ ← Remember to halve the coefficient of x

$= (x + 3)^2 - 14$ and subtract $(-3)^2$ to compensate

The turning point occurs when $(x + 3)^2 = 0$, i.e. when $x = -3$

When $x = -3$, $y = (-3 + 3)^2 - 14 = 0 - 14 = -14$

So the coordinates of the turning point is $(-3, -14)$

Exercise B

1. By writing the following in the form $y = (x + a)^2 + b$, where a and b are integers, write down the coordinates of the turning point of the curve.

(a) $y = x^2 - 8x + 20$ (b) $y = x^2 - 10x - 1$ (c) $y = x^2 + 4x - 6$

SOLVING QUADRATIC EQUATIONS	
What you should know from GSCE	**Video links to help**
• To solve quadratic equations by factorising when the coefficient of x^2 is 1 **•To solve quadratic equations by factorising, completing the square and the quadratic formula**	https://app.mymaths.co.uk/1784-lesson/quadratic-equations-1 https://app.mymaths.co.uk/192-lesson/quadratic-equations-2 https://app.mymaths.co.uk/193-lesson/completing-the-square https://app.mymaths.co.uk/194-lesson/quadratic-formula https://app.mymaths.co.uk/189-lesson/quadratic-equations-fractions https://corbettmaths.com/2013/05/03/solving-quadratics-by-factorising/ https://corbettmaths.com/2013/04/24/quadratic-formula/ https://corbettmaths.com/2013/12/29/completing-the-square-video-10/ https://corbettmaths.com/2015/03/19/deriving-the-quadratic-formula/

Solving a Quadratic by factoring

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is *b* and whose products is *ac*.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1 : Solve $x^2 - 3x + 2 = 0$

Factorise $(x-1)(x-2) = 0$
Either $(x-1) = 0$ or $(x-2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x-2) = 0$
Either $x = 0$ or $(x-2) = 0$
So $x = 0$ or $x = 2$

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.
So $(3x + 4) = 0$ or $(3x - 4) = 0$	**2** When two values multiply to make zero, at least one of the values must be zero.
$x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)
So $2x^2 - 8x + 3x - 12 = 0$	**2** Rewrite the b term ($-5x$) using these two factors.
$2x(x - 4) + 3(x - 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x - 4)(2x + 3) = 0$	**4** $(x - 4)$ is a factor of both terms.
So $(x - 4) = 0$ or $(2x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
$x = 4$ or $x = -\frac{3}{2}$	**6** Solve these two equations.

Exercise A

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving a Quadratic by completing the square

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 1 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$

$(x + 3)^2 - 9 + 4 = 0$

$(x + 3)^2 - 5 = 0$

$(x + 3)^2 = 5$

$x + 3 = \pm\sqrt{5}$

$x = \pm\sqrt{5} - 3$

So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$

1 Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$

2 Simplify.

3 Rearrange the equation to work out x. First, add 5 to both sides.

4 Square root both sides. Remember that the square root of a value gives two answers.

5 Subtract 3 from both sides to solve the equation.

6 Write down both solutions.

Example 2 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$

$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$

$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$

$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$

$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$

$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$

$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$

$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$

$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$

So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$

1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$

2 Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x+\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$

3 Expand the square brackets.

4 Simplify.

(continued on next page)

5 Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.

6 Divide both sides by 2.

7 Square root both sides. Remember that the square root of a value gives two answers.

8 Add $\frac{7}{4}$ to both sides.

9 Write down both the solutions.

Exercise B

1 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

2 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving a Quadratics by using the formula

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 1 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1$, $b = 6$, $c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 Identify a, b and c and write down the formula.

Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

2 Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.

$$x = \frac{-6 \pm \sqrt{20}}{2}$$

3 Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.

$$x = \frac{-6 \pm 2\sqrt{5}}{2}$$

4 Simplify $\sqrt{20}$.

$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

$$x = -3 \pm \sqrt{5}$$

5 Simplify by dividing numerator and denominator by 2.

So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$

6 Write down both the solutions.

Example 2 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3$, $b = -7$, $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 Identify a, b and c, making sure you get the signs right and write down the formula.

Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$$

2 Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.

$$x = \frac{7 \pm \sqrt{73}}{6}$$

3 Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.

So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$

4 Write down both the solutions.

Exercise C

1 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

2 Solve the equation $x^2 - 7x + 2 = 2$

Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

3 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

Challenge

Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

SOLVING LINEAR EQUATIONS	
What you should know from GSCE	**Video links to help**
•Solve linear equations including equations that involve multiple brackets and algebraic fractions	https://app.mymaths.co.uk/1734-lesson/equations-4-brackets https://app.mymaths.co.uk/1735-lesson/equations-5-fractions https://corbettmaths.com/2012/08/24/solving-equations-with-letters-on-both-sides/ https://corbettmaths.com/2013/05/19/equations-cross-multiplication/ https://corbettmaths.com/2013/05/25/algebraic-equations/ https://corbettmaths.com/2015/12/07/equations-involving-algebraic-fractions-advanced/

Key Points

When solving an equation whatever you do to one side must also be done to the other.
You may

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, collect all the letters onto the same side of the equation.

If the equation contains brackets, you often start by expanding the brackets.

A linear equation contains only numbers and terms in x. (Not x^2 or x^3 or $\frac{1}{x}$ etc)

Examples

Example 1: Solve the equation $64 - 3x = 25$

Solution: There are various ways to solve this equation. $64 - 3x = 25$

Step 1: Add $3x$ to both sides (so that the x term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 - 2x$.

Solution:

$$6x + 7 = 5 - 2x.$$

Step 1: Begin by adding $2x$ to both sides (to ensure that the x terms are together on the same side)

$$8x + 7 = 5$$

Step 2: Subtract 7 from each side:

$$8x = -2$$

Step 3: Divide each side by 8:

$$x = -\tfrac{1}{4}$$

Example 3: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: (taking care of the negative signs) $\quad 6x - 4 = 20 - 3x - 6$

Step 2: Simplify the right hand side: $\quad 6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $\quad 9x - 4 = 14$

Step 4: Add 4: $\quad 9x = 18$

Step 5: Divide by 9: $\quad x = 2$

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\frac{y}{2} + 5 = 11$

$$\frac{y}{2} + 5 = 11$$

Solution: Step 1: Multiply through by 2 (the denominator in the fraction): $\quad y + 10 = 22$

Step 2: Subtract 10: $\quad y = 12$

Example 5: Solve the equation $\frac{1}{3}(2x+1) = 5$

$$\frac{1}{3}(2x+1) = 5$$

Solution: Step 1: Multiply by 3 (to remove the fraction) $\quad 2x + 1 = 15$

Step 2: Subtract 1 from each side $\quad 2x = 14$

Step 3: Divide by 2 $\quad x = 7$

Example 6: Solve the equation $\frac{x+1}{4}+\frac{x+2}{5}=2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator

$$\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$$

Step 3: Simplify the left hand side:

$$\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$$

$5(x+1)+4(x+2)=40$

Step 4: Multiply out the brackets: $5x+5+4x+8=40$

Step 5: Simplify the equation: $9x+13=40$

Step 6: Subtract 13 $9x=27$

Step 7: Divide by 9: $x=3$

Exercise A – Solve the following equations

1) $2x+5=19$	2) $5x-2=13$	3) $11-4x=5$
4) $5-7x=-9$	5) $11+3x=8-2x$	6) $7x+2=4x-5$

Exercise B – Solve the following equations with brackets

1) $5(2x-4)=4$	2) $4(2-x)=3(x-9)$
3) $8-(x+3)=4$	4) $14-3(2x+3)=2$

Exercise C – Solve the following equations with fractions

1) $\frac{1}{2}(x+3)=5$

2) $\frac{2x}{3}-1=\frac{x}{3}+4$

3) $\frac{y}{4}+3=5-\frac{y}{3}$

4) $\frac{x-2}{7}=2+\frac{3-x}{14}$

5) $\frac{7x-1}{2}=13-x$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

Forming and solving Equations

Example Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.

Therefore

$$n+(n+1)+(n+2)=96$$
$$3n+3=96$$
$$3n=93$$
$$n=31$$

So the numbers are 31, 32 and 33.

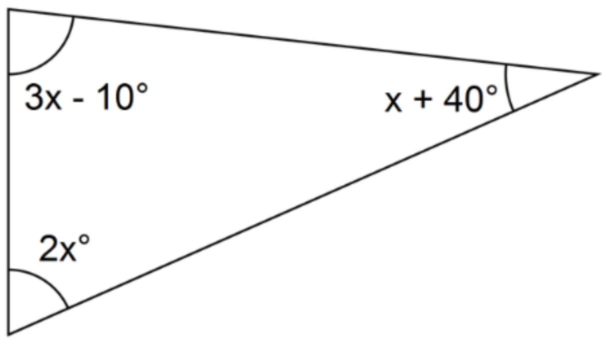
Challenge Questions

1) Find 3 consecutive even numbers so that their sum is 108.
2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

Problem Solving

Task 1

What type of triangle is this?
(Not drawn to scale)



Task 2

Adam, Bella and Carl collect tokens in a game.
Bella has 10 more tokens than Adam.
Carl has 8 more tokens than Bella.

Altogether they have 91 tokens.

How many tokens does Adam have?

SIMULTANEOUS EQUATIONS	
What you should know from GSCE	**Video links to help**
• To set up and solve linear simultaneous equations **•To solve linear and quadratic simultaneous equations** **•To know that the points of intersection of a curve and a straight line are the solutions to the simultaneous equations for the line and the curve**	https://app.mymaths.co.uk/196-lesson/simultaneous-equations-1 https://app.mymaths.co.uk/197-lesson/simultaneous-equations-2 https://app.mymaths.co.uk/198-lesson/simultaneous-equations-3 https://app.mymaths.co.uk/199-lesson/simultaneous-negatives https://app.mymaths.co.uk/195-lesson/quadratic-simultaneous-equs https://corbettmaths.com/2013/03/05/simultaneous-equations-elimination-method/ https://corbettmaths.com/2013/05/07/solving-simultaneous-equations-by-substitution/ https://corbettmaths.com/2013/05/07/simultaneous-equations-linear-and-quadratic/

Solving Linear Simultaneous Equations

•Two equations are simultaneous when they are both true at the same time.

•Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.

•Make sure that the coefficient of one of the unknowns is the same in both equations.

•Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$$\begin{array}{rl} & 3x + y = 5 \\ - & x + y = 1 \\ \hline & 2x = 4 \end{array}$$

So $x = 2$

1 Subtract the second equation from the first equation to eliminate the y term.

Using $x + y = 1$

$2 + y = 1$

So $y = -1$

2 To find the value of y, substitute $x = 2$ into one of the original equations.

Check:

equation 1: $3 \times 2 + (-1) = 5$ YES

equation 2: $2 + (-1) = 1$ YES

3 Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$$\begin{array}{rl} & x + 2y = 13 \\ + & 5x - 2y = \ \ 5 \\ \hline & 6x \qquad = 18 \end{array}$$

So $x = 3$

1 Add the two equations together to eliminate the y term.

Using $x + 2y = 13$

$3 + 2y = 13$

So $y = 5$

2 To find the value of y, substitute $x = 3$ into one of the original equations.

Check:

equation 1: $3 + 2 \times 5 = 13$ YES

equation 2: $5 \times 3 - 2 \times 5 = 5$ YES

3 Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$$\begin{array}{ll} (2x + 3y = 2) \times 4 \rightarrow & 8x + 12y = \ \ 8 \\ (5x + 4y = 12) \times 3 \rightarrow & 15x + 12y = 36 \\ \cline{2-2} & 7x \qquad = 28 \end{array}$$

So $x = 4$

1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.

Using $2x + 3y = 2$

$2 \times 4 + 3y = 2$

So $y = -2$

2 To find the value of y, substitute $x = 4$ into one of the original equations.

Check:

equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES

equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES

3 Substitute the values of x and y into both equations to check your answers.

Solving Linear and Quadratic Simultaneous Equations

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + 2x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2,\ y = 3$ and $x = -3, y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \dfrac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\dfrac{5-3y}{2}\right)y = 12$	**2** Substitute $\dfrac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \dfrac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$	**3** Expand the brackets and simplify.
$y^2 + 5y - 24 = 0$ $(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	**6** To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5,\ y = -8$ and $x = -2, y = 3$	
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Exercise A – Solving the following pairs of linear simultaneous equations

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

Exercise B – Solve these pairs of simultaneous equation

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Challenge Questions

1 $x^2 + 3xy + 5y^2 = 15$
$x - y = 1$

2 $xy + x^2 + y^2 = 7$
$x - 3y = 5$

3 $x^2 + 3xy + 5y^2 = 5$
$x - 2y = 1$

4 $4x^2 - 4xy - 3y^2 = 20$
$2x - 3y = 10$

5 $x^2 - y^2 = 11$
$x - y = 11$

6 $\frac{12}{x} + \frac{1}{y} = 3$
$x + y = 7$

Problem Solving

Task 1 – forming and solving

1. Laura and Dora each have some stickers. Altogether they have 87. Laura has 9 more than Dora. Create and solve simultaneous equations to show this information and to find the number of stickers that each has.

2. Luca has some 10 pence pieces and some 5 pence pieces. Altogether he has 40 coins and £3.15. How many of each coin does he have?

3. A rectangular sheet of paper has perimeter 60cm. When it is folded in half along its longer line of symmetry, the perimeter of the rectangle which this creates is 49cm. What are the dimensions of the original shape?

4. Seema makes and sells pencil cases and makeup bags. It takes her 10 hours to make 5 of each. It takes her 10 hours 30 minutes to make 4 pencil cases and 6 makeup bags. How long does it take her to make 1 pencil case and how long does it take her to make 1 makeup bag?

Task 2

Matchless

There is a particular value of x, and a value of y to go with it, which make all five expressions equal in value.
Can you find that x, y pair?

$2x + 3y - 20$

$5x - 2y + 38$

$4x + 5y - 72$

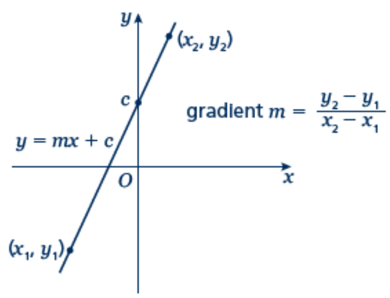
$x - 4y + 108$

$3x - y + 39$

STRAIGHT LINE GRAPHS	
What you should know from GCSE	**Video links to help**
•To identify the main features of straight-line graphs and use them to sketch graphs **•To sketch graphs from linear equations in the form of y = mx + c** **•To find the equation of a straight line using gradient and points on the line** **•To find the equation of a straight line using the coordinates of two points on a line** **•To identify lines that are parallel by considering their equations** **•To find the equation of a line parallel to a given line (perhaps passing through a known point)** **•To identify and find equations of perpendicular lines** **•To find the equation of a tangent that touches a circle centred on the origin**	https://app.mymaths.co.uk/219-lesson/gradient-and-intercept https://app.mymaths.co.uk/220-lesson/y-mx-c https://app.mymaths.co.uk/221-lesson/equation-of-a-line-2 https://app.mymaths.co.uk/3270-lesson/equation-of-a-line-3 https://corbettmaths.com/2013/05/28/gradient-between-two-points/ https://corbettmaths.com/2013/05/29/finding-the-equation-of-a-straight-line/ https://corbettmaths.com/2013/05/29/finding-the-equation-passing-through-two-points/ https://corbettmaths.com/2013/06/06/graphs-parallel-lines/ https://corbettmaths.com/2013/06/06/perpendicular-lines-2/

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$

So $y = -\frac{1}{2}x + 3$

$\frac{1}{2}x + y - 3 = 0$

$x + 2y - 6 = 0$

1 A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation.

2 Rearrange the equation so all the terms are on one side and 0 is on the other side.

3 Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$

$3y = 2x - 4$

$y = \frac{2}{3}x - \frac{4}{3}$

Gradient $= m = \frac{2}{3}$

y-intercept $= c = -\frac{4}{3}$

1 Make y the subject of the equation.

2 Divide all the terms by three to get the equation in the form $y = \ldots$

3 In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$

$y = 3x + c$

$13 = 3 \times 5 + c$

$13 = 15 + c$

$c = -2$

$y = 3x - 2$

1 Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.

2 Substitute the coordinates $x = 5$ and $y = 13$ into the equation.

3 Simplify and solve the equation.

4 Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$

$y = \frac{1}{2}x + c$

$4 = \frac{1}{2} \times 2 + c$

$c = 3$

$y = \frac{1}{2}x + 3$

1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.

2 Substitute the gradient into the equation of a straight line $y = mx + c$.

3 Substitute the coordinates of either point into the equation.

4 Simplify and solve the equation.

5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Exercise A

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

Hint
Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7 **b** gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4 **d** gradient –1.2, y-intercept –2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

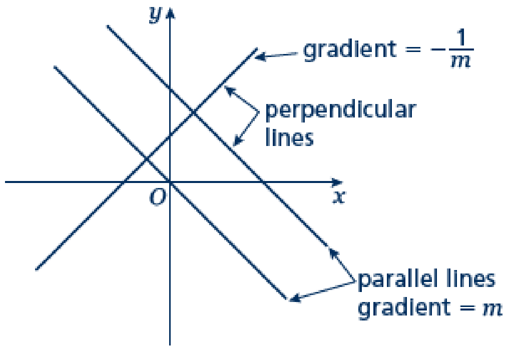
6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Parallel and Perpendicular Lines

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1
Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$	**1** As the lines are parallel they have the same gradient.
$y = 2x + c$	**2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2
Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	**2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	**3** Substitute the coordinates $(-2, 5)$ into the equation $y = -\frac{1}{2}x + c$
$5 = 1 + c$ $c = 4$	**4** Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and $(9, -1)$. Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint $= \left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Exercise B

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ $(3, 2)$ **b** $y = 3 - 2x$ $(1, 3)$

c $2x + 4y + 3 = 0$ $(6, -3)$ **d** $2y - 3x + 2 = 0$ $(8, 20)$

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point $(-5, 3)$.

Hint
If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ $(4, 0)$ **b** $y = -\frac{1}{3}x + \frac{1}{2}$ $(2, 13)$

c $x - 4y - 4 = 0$ $(5, 15)$ **d** $5y + 2x - 5 = 0$ $(6, 7)$

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a $(4, 3)$, $(-2, -9)$ **b** $(0, 3)$, $(-10, 8)$

Challenge Questions

1 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

2 The straight line $\mathbf{L}_1$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of $\mathbf{L}_1$ in the form $ax + by + c = 0$

The line $\mathbf{L}_2$ is parallel to the line $\mathbf{L}_1$ and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of $\mathbf{L}_2$ in the form $ax + by + c = 0$

Problem Solving with straight line graphs

Task 1

Here are some equations of straight lines:

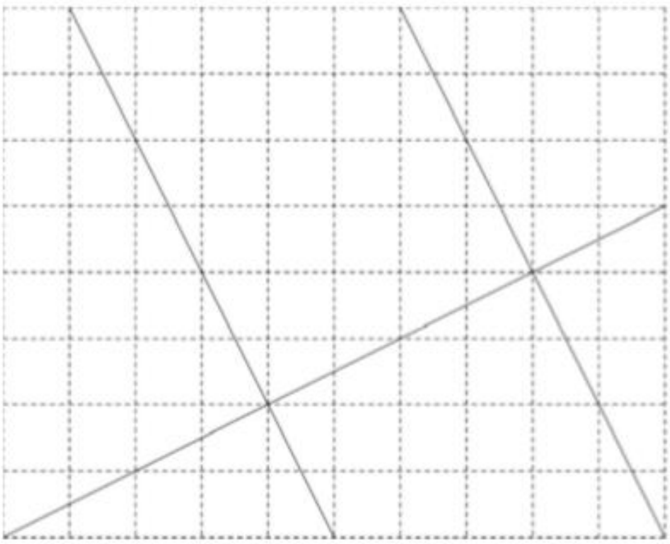
$y+2x=8$	$2y+\frac{1}{2}x+1=0$	$2y+x=1$	$y=x-4$	$y=2(x-1)$
$2y=x-4$	$y+2x+2=0$	$y=\frac{1}{2}x+2$	$y=4-x$	$2y=4-x$

1. Which four lines form the four sides of a rectangle?

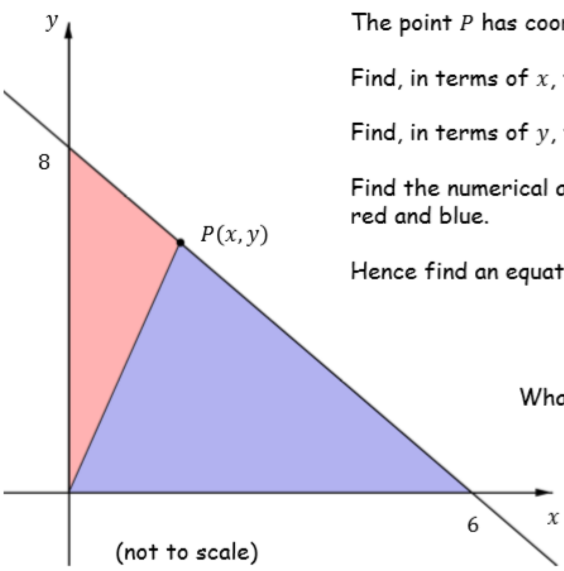
   Explain your reasoning carefully.

2. Complete the drawing below to show the four lines and the x- and y-axes.

   Label the lines clearly.



Task 2



The point P has coordinates (x, y)

Find, in terms of x, the area of the red triangle.

Find, in terms of y, the area of the blue triangle.

Find the numerical area of the triangle which is red and blue.

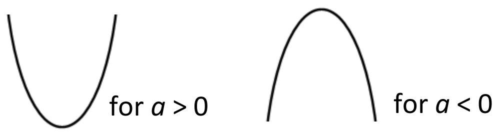
Hence find an equation linking x and y.

What is the equation of the line?

QUADRATIC GRAPHS	
What you should know from GSCE	**Video links to help**
•To be able to identify and plot graphs of quadratic functions i.e. parabolas **•To find roots of quadratic equations from the x-intercept of the parabola of the quadratic equation that defines the graph** **•To know the features of graphs of quadratic equations** **•To be able to sketch parabolas**	https://app.mymaths.co.uk/215-lesson/plotting-graphs-3-quadratics https://app.mymaths.co.uk/3267-lesson/properties-of-quadratics https://app.mymaths.co.uk/3272-lesson/sketching-quadratic-graphs-1 https://app.mymaths.co.uk/226-lesson/sketching-quadratic-graphs-2 https://corbettmaths.com/2013/06/23/drawing-quadratics/ https://corbettmaths.com/2013/06/22/sketching-quadratics/ https://corbettmaths.com/2017/09/25/quadratic-graphs-completing-the-square/

Key Points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.



- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

Example 1 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$
So the graph intersects the y-axis at $(0, -6)$

1 Find where the graph intersects the y-axis by substituting $x = 0$.

When $y = 0$, $x^2 - x - 6 = 0$

2 Find where the graph intersects the x-axis by substituting $y = 0$.

$(x + 2)(x - 3) = 0$

3 Solve the equation by factorising.

$x = -2$ or $x = 3$

4 Solve $(x + 2) = 0$ and $(x - 3) = 0$.

So,
the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$

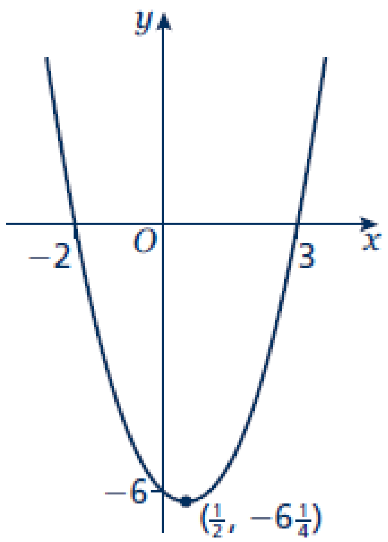
5 $a = 1$ which is greater than zero, so the graph has the shape:

$$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$$
$$= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$$

6 To find the turning point, complete the square.

When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$

7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.



Exercise A

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 + x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

Challenge Questions

1 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

2 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

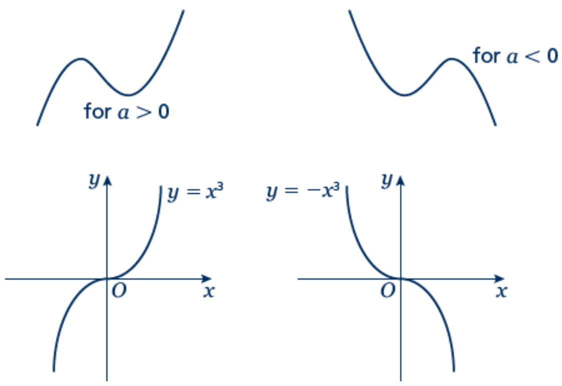
a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

3 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

OTHER GRAPHS	
What you should know from GCSE	**Video links to help**
•To be able to work fluently with cubic polynomials and their graphs •To be able to sketch cubic graphs •To work fluently to calculate reciprocals of numbers and plot functions involving reciprocals •To identify hyperbolas and match them to their equations •To plot and sketch graphs from given functions	https://app.mymaths.co.uk/3266-lesson/sketching-cubic-graphs https://app.mymaths.co.uk/223-lesson/reciprocals https://app.mymaths.co.uk/222-lesson/plotting-graphs-4-non-linear https://corbettmaths.com/2016/08/07/cubic-graphs/ https://corbettmaths.com/2013/10/24/reciprocal-graphs/

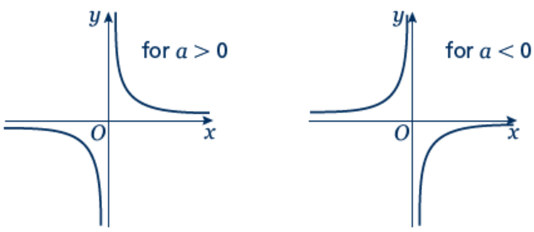
Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



special case: $a = 1$ special case: $a = -1$

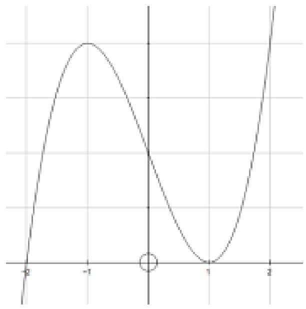
- The graph of a reciprocal function of the form $y = \frac{a}{x}$ has one of the shapes shown here.



- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double (repeated) root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double (repeated) root at $x = 3$.
- When there is a double (repeated) root, this is one of the turning points of a cubic function.

Repeated factors

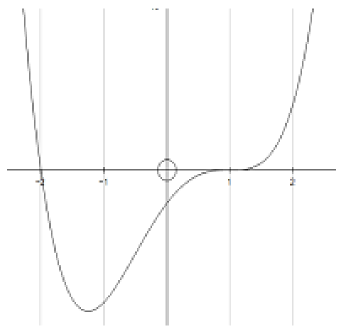
Suppose you want to sketch $y = (x - 1)^2(x + 2)$.
You know there is an intercept at $x = -2$.
At $x = 1$ the graph *touches* the axes, as if it were the graph of $y = (x - 1)^2$ there.



[More precisely, it is very like $y = 3(x - 1)^2$ there. That is because, close to $x = 1$, the $(x - 1)^2$ factor changes rapidly, while $(x + 2)$ remains close to 3.]

Likewise, the graph of $y = (x + 2)(x - 1)^3$ looks like $y = (x - 1)^3$ close to $x = 1$.

[Again, more precisely, it is very like $y = 3(x - 1)^3$ there.]



Examples

Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

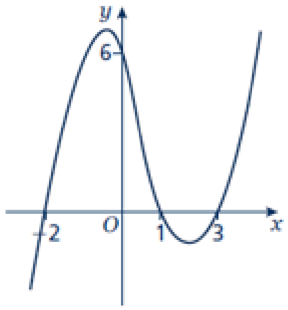
To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.

When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$
$= (-3) \times (-1) \times 2 = 6$
The graph intersects the y-axis at $(0, 6)$

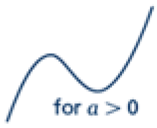
1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y).

When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$
So $x = 3$, $x = 1$ or $x = -2$
The graph intersects the x-axis at $(-2, 0)$, $(1, 0)$ and $(3, 0)$

2 Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$



3 Sketch the graph.
$a = 1 > 0$ so the graph has the shape:



Example 2 Sketch the graph of $y = (x + 2)^2(x - 1)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.

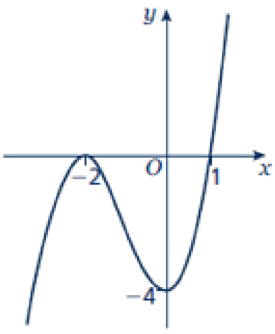
When $x = 0$, $y = (0 + 2)^2(0 - 1)$
$= 2^2 \times (-1) = -4$
The graph intersects the y-axis at $(0, -4)$

1 Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$.

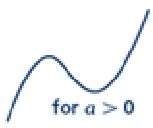
When $y = 0$, $(x + 2)^2(x - 1) = 0$
So $x = -2$ or $x = 1$

2 Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$

$(-2, 0)$ is a turning point as $x = -2$ is a double root.
The graph crosses the x-axis at $(1, 0)$



3 $a = 1 > 0$ so the graph has the shape:



Exercise A

1 Here are six equations.

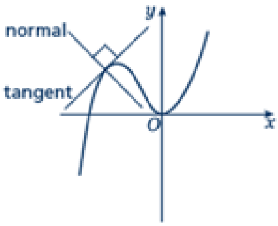
A $y = \frac{5}{x}$ **B** $y = x^2 + 3x - 10$ **C** $y = x^3 + 3x^2$

D $y = 1 - 3x^2 - x^3$ **E** $y = x^3 - 3x^2 - 1$ **F** $x + y = 5$

> **Hint**
>
> Find where each of the cubic equations cross the y-axis.

Here are six graphs.

i



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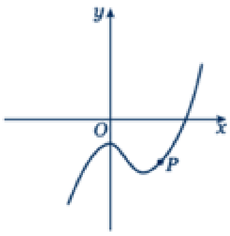

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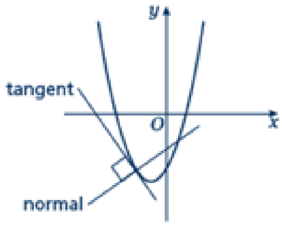

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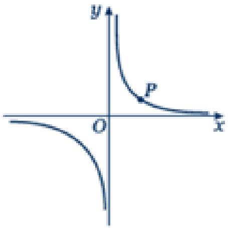

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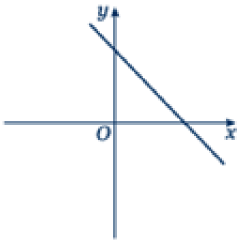

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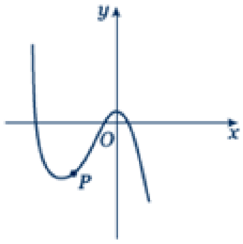


a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

Sketch the following graphs

2 $y = 2x^3$

3 $y = x(x - 2)(x + 2)$

4 $y = (x + 1)(x + 4)(x - 3)$

5 $y = (x + 1)(x - 2)(1 - x)$

6 $y = (x - 3)^2(x + 1)$

7 $y = (x - 1)^2(x - 2)$

8 $y = \frac{3}{x}$

> **Hint:** Look at the shape of $y = \frac{a}{x}$ in the second key point.

9 $y = -\frac{2}{x}$

Challenge Questions

1 Sketch the graph of $y = \frac{1}{x+2}$

2 Sketch the graph of $y = \frac{1}{x-1}$

INEQUALITIES	
What you should know from GCSE	**Video links to help**
•Understanding and interpreting inequalities and using the correct symbols to express inequalities **•To use a number line and set notation to represent an inequality** **•Solving linear inequalities in one variable and representing the solution set on a number line** **•To solve quadratic inequalities** **•To solve (several) linear inequalities in two variables and represent the solution set on a graph**	https://app.mymaths.co.uk/1740-lesson/inequalities-and-intervals https://app.mymaths.co.uk/232-lesson/inequations https://app.mymaths.co.uk/233-lesson/negative-inequations https://app.mymaths.co.uk/234-lesson/shading-inequalities https://app.mymaths.co.uk/235-lesson/quadratic-inequalities https://corbettmaths.com/2013/05/18/inequalities/ https://corbettmaths.com/2013/05/07/solving-inequalities-one-sign-corbettmaths/ https://corbettmaths.com/2013/05/12/solving-inequalities-two-signs/ https://corbettmaths.com/2016/08/07/quadratic-inequalities/ https://corbettmaths.com/2013/05/27/graphical-inequalities-part-1/ https://corbettmaths.com/2013/05/27/graphical-inequalities-part-2/

Linear Inequalities – Key Points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \le 4x < 16$

$$-8 \le 4x < 16$$
$$-2 \le x < 4$$

Divide all three terms by 4.

Example 2 Solve $2x - 5 < 7$

$$2x - 5 < 7$$
$$2x < 12$$
$$x < 6$$

1 Add 5 to both sides.
2 Divide both sides by 2.

Example 3 Solve $2 - 5x \geq -8$

$$2 - 5x \geq -8$$
$$-5x \geq -10$$
$$x \leq 2$$

1 Subtract 2 from both sides.
2 Divide both sides by −5.
Remember to reverse the inequality when dividing by a negative number.

Example 4 Solve $4(x - 2) > 3(9 - x)$

$$4(x - 2) > 3(9 - x)$$
$$4x - 8 > 27 - 3x$$
$$7x - 8 > 27$$
$$7x > 35$$
$$x > 5$$

1 Expand the brackets.
2 Add $3x$ to both sides.
3 Add 8 to both sides.
4 Divide both sides by 7.

Exercise A

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Challenge Question

1 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

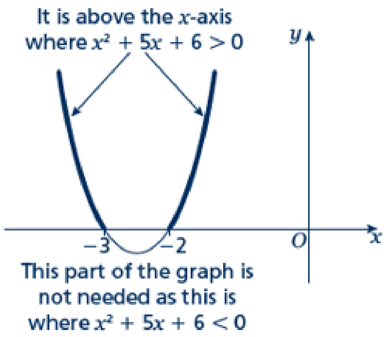
Quadratic Inequalities – Key Points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ or $x = -2$

1 Solve the quadratic equation by factorising.



2 Sketch the graph of $y = (x + 3)(x + 2)$

3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$

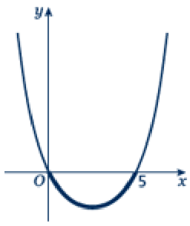
4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

$x < -3$ or $x > -2$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0$ or $x = 5$

1 Solve the quadratic equation by factorising.



2 Sketch the graph of $y = x(x - 5)$

3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$

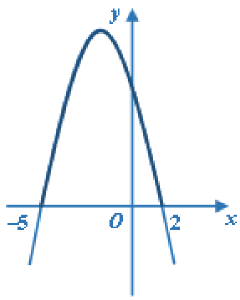
4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

$0 \leq x \leq 5$

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$
$(-x + 2)(x + 5) = 0$
$x = 2$ or $x = -5$

1 Solve the quadratic equation by factorising.



2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$

3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$

4 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

$-5 \leq x \leq 2$

Exercise B

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

More difficult…

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Problem Solving

Task 1

Chris runs a barber's shop. It costs him £20 per day to cover his expenses and he charges £4 for every hair cut.

(a) Explain why his profit for any day is £$(4x - 20)$, where x is the number of haircuts in that day.

He hopes to make at least £50 profit per day, but does not intend to make more than £120 profit.

(b) Write down an inequality to describe this situation.

(c) Solve the inequality.

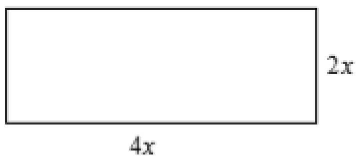
Task 2

(a) Write down an expression, in terms of x, for the area, A, of the rectangle

(b) If the area, A, of the rectangle satisfies the inequality

$$32 \leq A \leq 200,$$

write down an inequality for x and solve it.



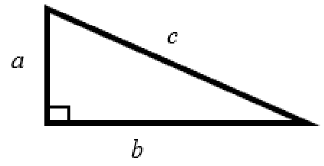
(c) What is the maximum length of the rectangle?

(d) What is the minimum width of the rectangle?

TRIGONOMETRY	
What you should know from GSCE	**Video links to help**
•Know and use Pythagoras' theorem to find any missing length in a right-angled triangle •To use Pythagoras' theorem to show whether a triangle is right-angled or not •To apply Pythagoras' theorem to 2D and 3D problems •Use the trigonometric ratios given by the sine, cosine and tangent functions to find unknown lengths and angles in 2D right-angled triangles •Know the exact ratios given by sine and cosine of 0, 30, 45, 60 and 90 degrees and the exact ratios given by the tangent function for 0, 30, 45 and 60 degrees •To use the sine, cosine and sine area rules to solve problems relating to unknown sides, angles and areas in non-right-angled triangles •To identify and plot trigonometric graphs	https://app.mymaths.co.uk/300-lesson/pythagoras-theorem https://app.mymaths.co.uk/301-lesson/pythagoras-3d https://app.mymaths.co.uk/321-lesson/trig-missing-angles https://app.mymaths.co.uk/322-lesson/trig-missing-sides https://app.mymaths.co.uk/324-lesson/sine-rule https://app.mymaths.co.uk/325-lesson/cosine-rule-sides https://app.mymaths.co.uk/326-lesson/cosine-rule-angles https://app.mymaths.co.uk/327-lesson/trig-area-of-a-triangle https://app.mymaths.co.uk/329-lesson/sine-and-cosine-graphs https://app.mymaths.co.uk/330-lesson/tan-graphs https://corbettmaths.com/2012/08/19/pythagoras-video/ https://corbettmaths.com/2012/08/19/3d-pythagoras/ https://corbettmaths.com/2013/06/22/pythagoras-rectangles-and-isosceles-triangles/ https://corbettmaths.com/2013/03/30/trigonometry-missing-sides/ https://corbettmaths.com/2013/03/30/trigonometry-missing-angles/ https://corbettmaths.com/2013/05/03/sine-rule-missing-sides/ https://corbettmaths.com/2019/04/24/sine-rule-angles/ https://corbettmaths.com/2013/04/04/cosine-rule-missing-sides/ https://corbettmaths.com/2013/04/04/cosine-rule-missing-angles/ https://corbettmaths.com/2012/08/02/area-of-a-triangle-sinetrigonometry/ https://corbettmaths.com/2013/04/20/ysinx-graph/ https://corbettmaths.com/2013/05/07/cosine-graph/ https://corbettmaths.com/2013/05/12/tangent-graph/ https://corbettmaths.com/2013/04/20/exact-trigonometric-values/

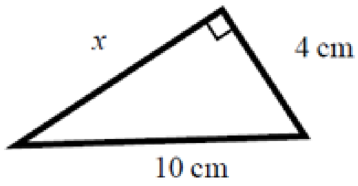
Pythagoras' Theorem

- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



Example

Example 1 Calculate the length x.
Give your answer in surd form.



$c^2 = a^2 + b^2$

$10^2 = x^2 + 4^2$
$100 = x^2 + 16$
$x^2 = 84$
$x = \sqrt{84}$
$x = 2\sqrt{21}$ cm

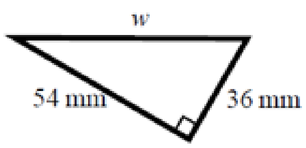
1 Always start by stating the formula for Pythagoras' theorem.
2 Substitute the values of a, b and c into the formula for Pythagoras' theorem.

3 Simplify the surd where possible and write the units in your answer.

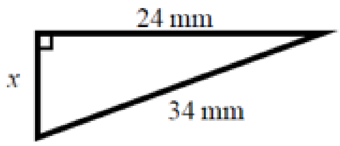
Exercise A

1 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

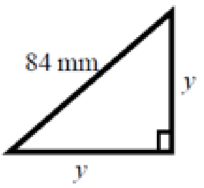
a



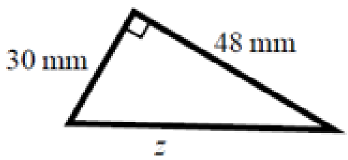
b



c



d



2 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

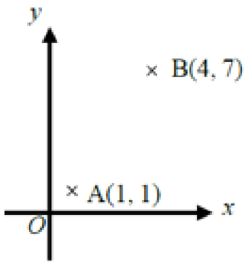
> **Hint**
>
> Draw a sketch of the rectangle.

Challenge Questions

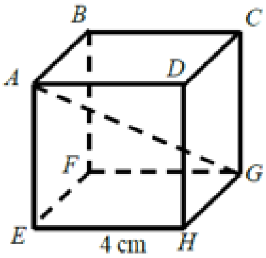
1 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

> **Hint**
>
> Draw a diagram using the information given in the question.

2 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form

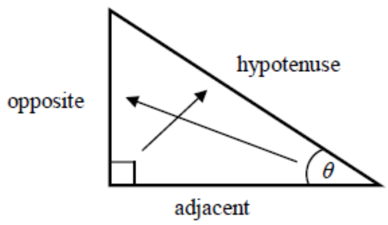


3 A cube has length 4 cm. Work out the length of the diagonal AG.
Give your answer in surd form.



Trigonometry – Right Angled Triangles

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle ϑ is called the opposite
  - the side next to the angle ϑ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle ϑ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle ϑ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle ϑ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

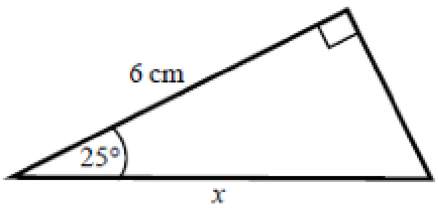
- The sine, cosine and tangent of some angles may be written exactly.

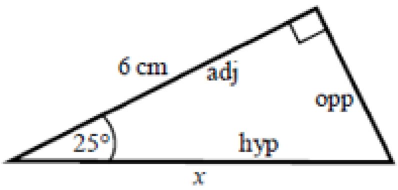
	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1

Calculate the length of side x.
Give your answer correct to 3 significant figures.





1 Always start by labelling the sides.

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

2 You are given the adjacent and the hypotenuse so use the cosine ratio.

$\cos 25° = \frac{6}{x}$

3 Substitute the sides and angle into the cosine ratio.

$x = \frac{6}{\cos 25°}$

4 Rearrange to make x the subject.

$x = 6.620\,267\,5\ldots$

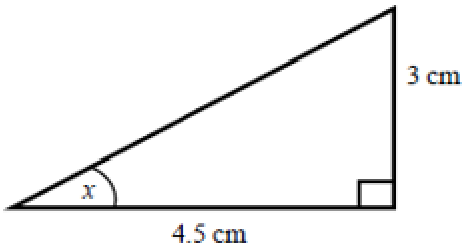
5 Use your calculator to work out $6 \div \cos 25°$.

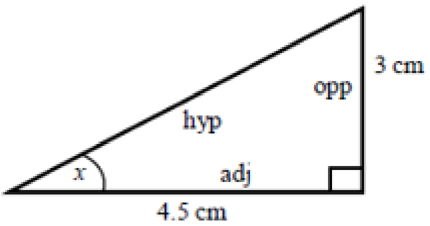
$x = 6.62$ cm

6 Round your answer to 3 significant figures and write the units in your answer.

Example 2

Calculate the size of angle x.
Give your answer correct to 3 significant figures.





1 Always start by labelling the sides.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

2 You are given the opposite and the adjacent so use the tangent ratio.

$$\tan x = \frac{3}{4.5}$$

3 Substitute the sides and angle into the tangent ratio.

$$x = \tan^{-1}\left(\frac{3}{4.5}\right)$$

4 Use $\tan^{-1}$ to find the angle.

$$x = 33.690\,067\,5\ldots$$

5 Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.

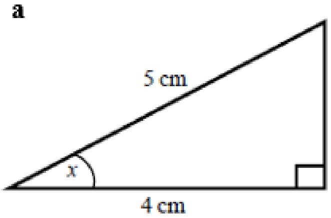
$$x = 33.7°$$

6 Round your answer to 3 significant figures and write the units in your answer.

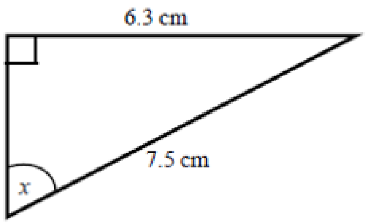
Exercise B

2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

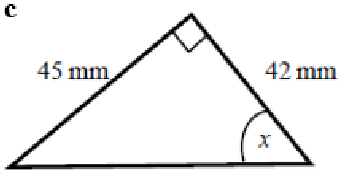
a



b

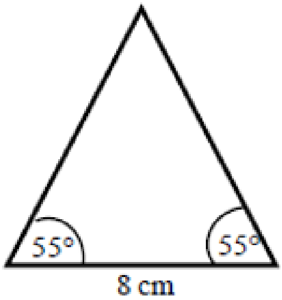


c



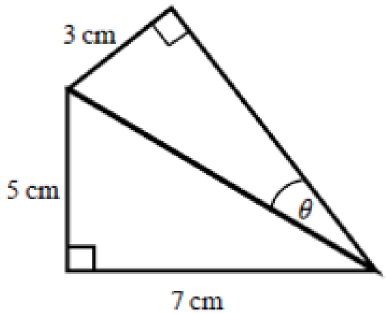
1 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

Hint: Split the triangle into two right-angled triangles.



2 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

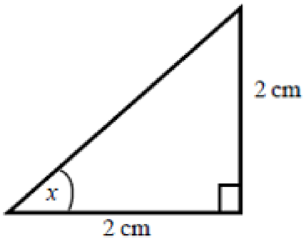
Hint: First work out the length of the common side to both triangles, leaving your answer in surd form.



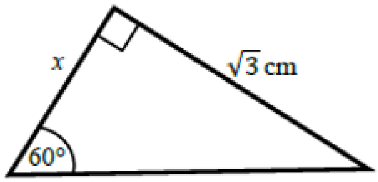
Challenge Questions

3 Find the exact value of x in each triangle.

a

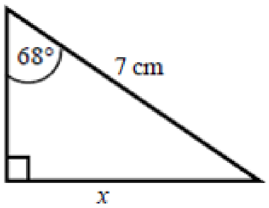


b

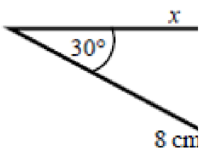


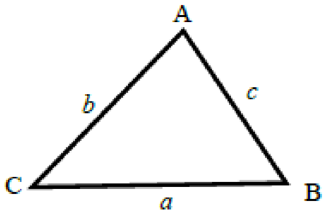
1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

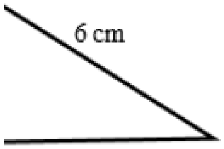
a



b







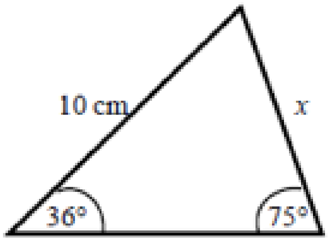
The Sine Rule – Key Points

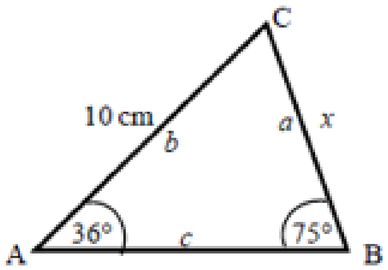
- *a* is the side opposite angle A.
  b is the side opposite angle B.

c is the side opposite angle C.

Example 1

Work out the length of side x.
Give your answer correct to 3 significant figures.





1 Always start by labelling the angles and sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

2 Write the sine rule to find the side.

$$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$$

3 Substitute the values a, b, A and B into the formula.

$$x = \frac{10 \times \sin 36°}{\sin 75°}$$

4 Rearrange to make x the subject.

$x = 6.09$ cm

5 Round your answer to 3 significant figures and write the units in your answer.

- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

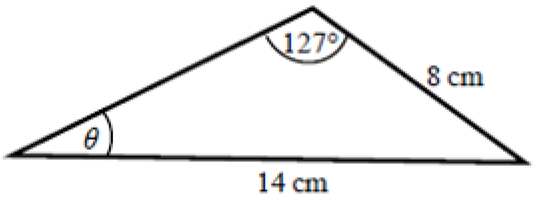
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

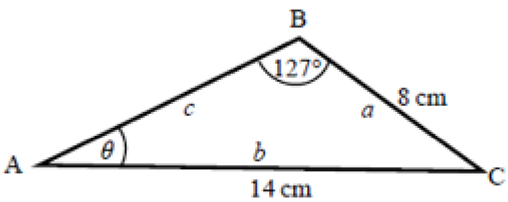
<u>Examples</u>

Exercise C

Example 2

Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$$

$$\sin \theta = \frac{8 \times \sin 127°}{14}$$

$$\theta = 27.2°$$

1 Always start by labelling the angles and sides.

2 Write the sine rule to find the angle.

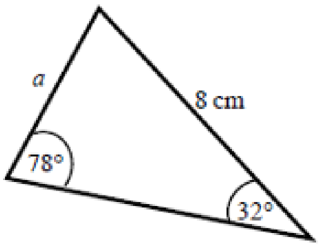
3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make $\sin \theta$ the subject.

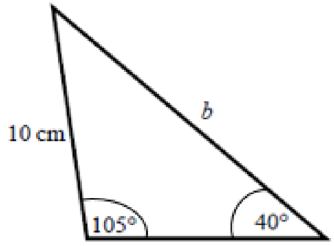
5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

1 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

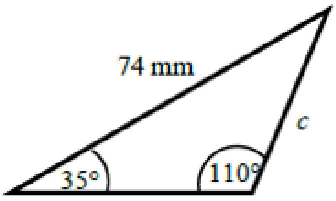
a



b

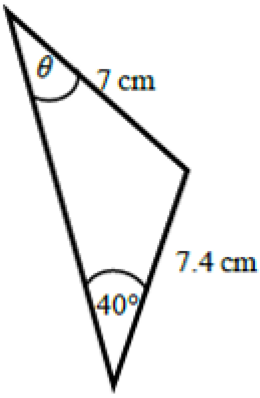


c

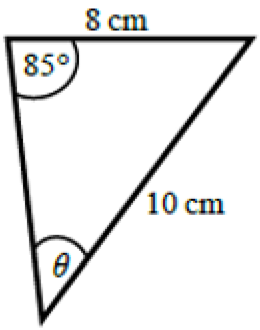


2 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

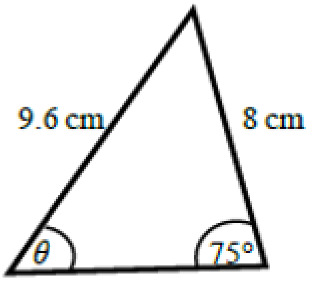
a



b



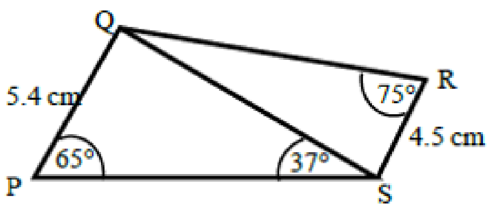
c



Challenge

1 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

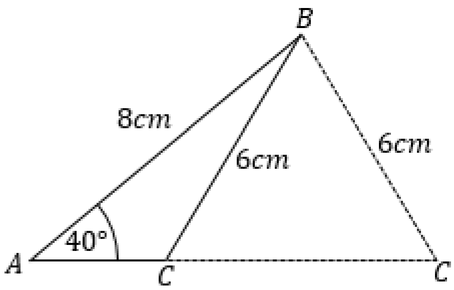
b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



Problem Solving

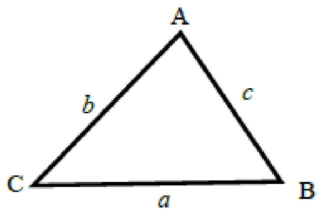
1. In triangle ABC, $AB = 8cm$, $BC = 6cm$ and angle $CAB = 40°$ (see diagram)

   Find the two possible sizes of angle BCA.



The Cosine Rule – Key points

- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

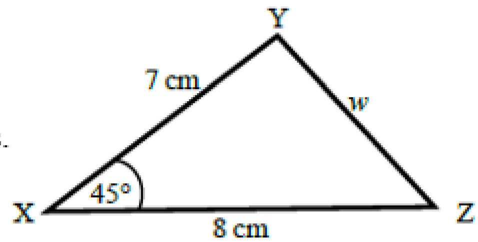


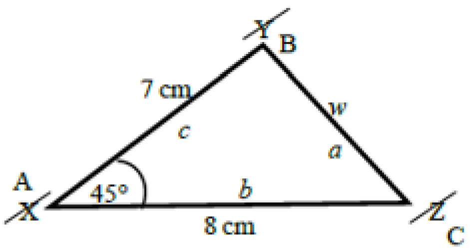
- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Examples

Example 1

Work out the length of side *w*.
Give your answer correct to 3 significant figures.





$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$

$w^2 = 33.804\,040\,51...$

$w = \sqrt{33.80404051}$

$w = 5.81$ cm

1 Always start by labelling the angles and sides.

2 Write the cosine rule to find the side.

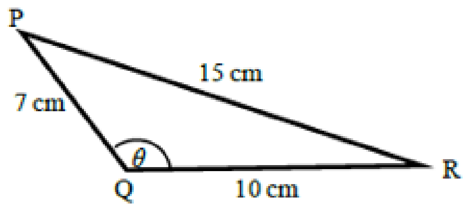
3 Substitute the values *a*, *b* and *A* into the formula.

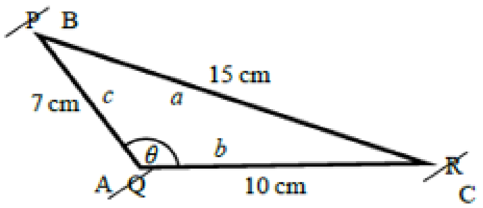
4 Use a calculator to find w^2 and then *w*.

5 Round your final answer to 3 significant figures and write the units in your answer.

Example 2

Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$$

$$\cos\theta = \frac{-76}{140}$$

$\theta = 122.878\,349...$

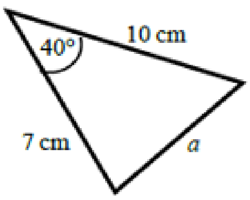
$\theta = 122.9°$

1. Always start by labelling the angles and sides.
2. Write the cosine rule to find the angle.
3. Substitute the values a, b and c into the formula.
4. Use $\cos^{-1}$ to find the angle.
5. Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
6. Round your answer to 1 decimal place and write the units in your answer.

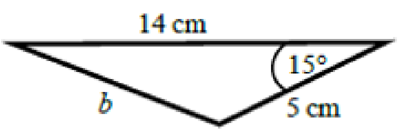
Exercise D

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

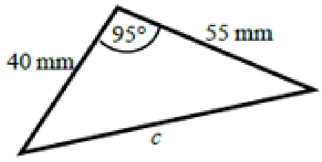
a



b

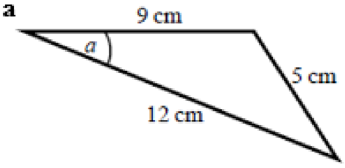


c

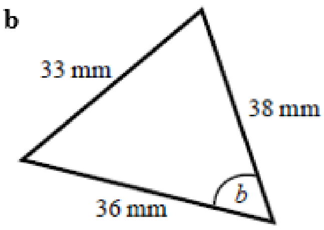


2 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

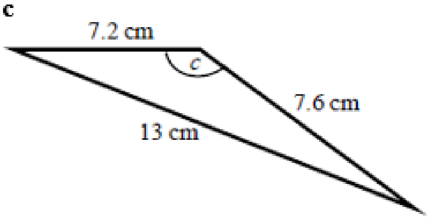
a



b



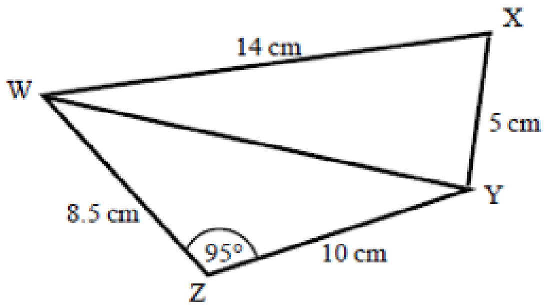
c



Challenge Question

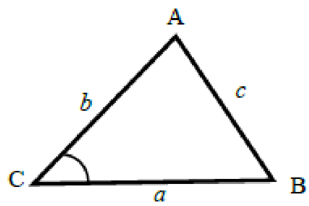
1 **a** Work out the length of WY.
Give your answer correct to 3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to 1 decimal place.



Area of a triangle – Key points

- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

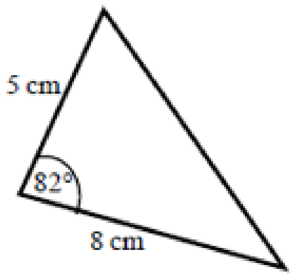


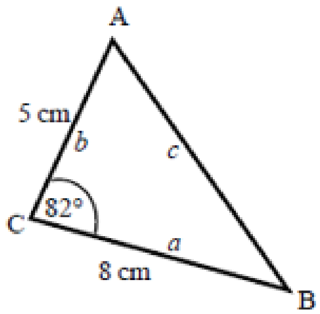
The area of the triangle is $\frac{1}{2}ab\sin C$.

Example

Example 1

Find the area of the triangle.





Area = $\frac{1}{2}ab\sin C$

Area = $\frac{1}{2} \times 8 \times 5 \times \sin 82°$

Area = 19.805 361...

Area = 19.8 cm^2

1 Always start by labelling the sides and angles of the triangle.

2 State the formula for the area of a triangle.

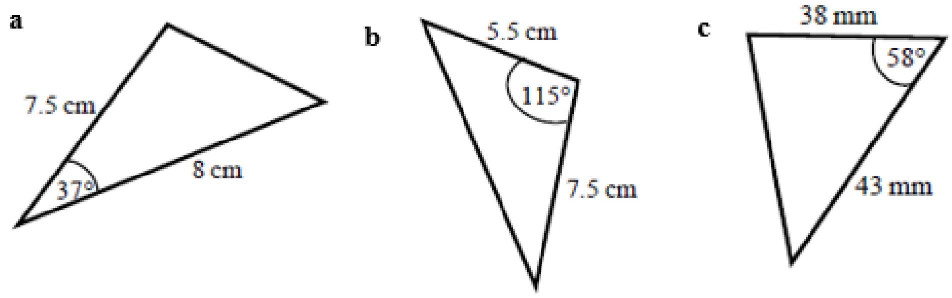
3 Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.

4 Use a calculator to find the area.

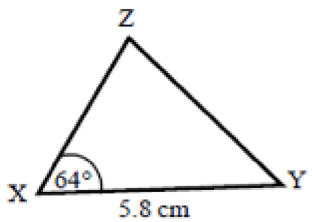
5 Round your answer to 3 significant figures and write the units in your

Exercise E

1 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

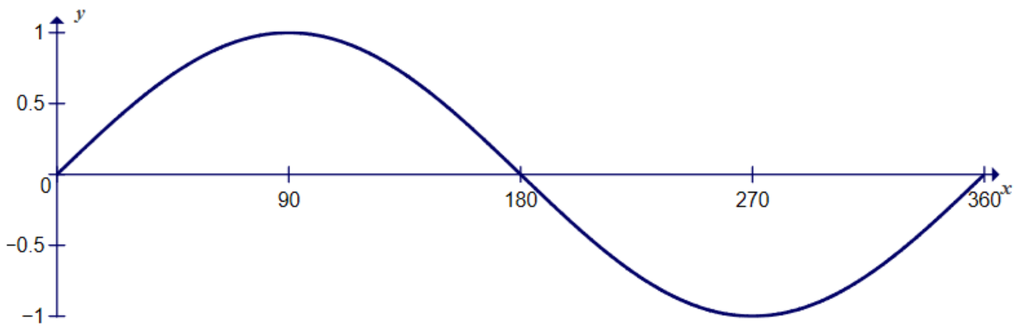


2 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.

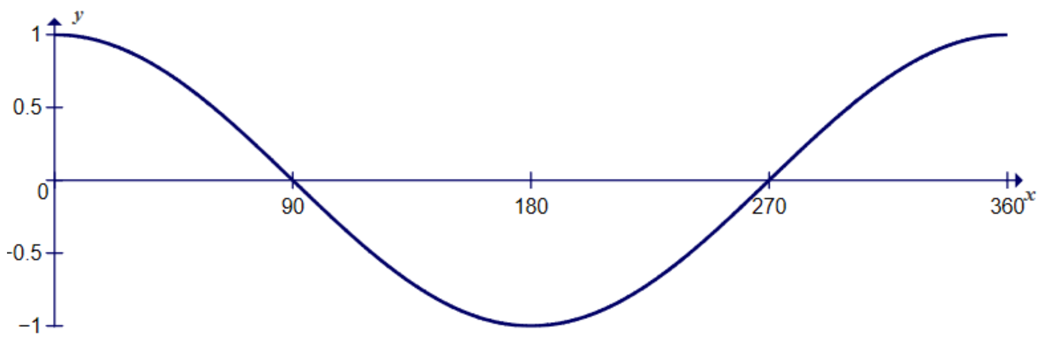


Trigonometric Graphs – Key points

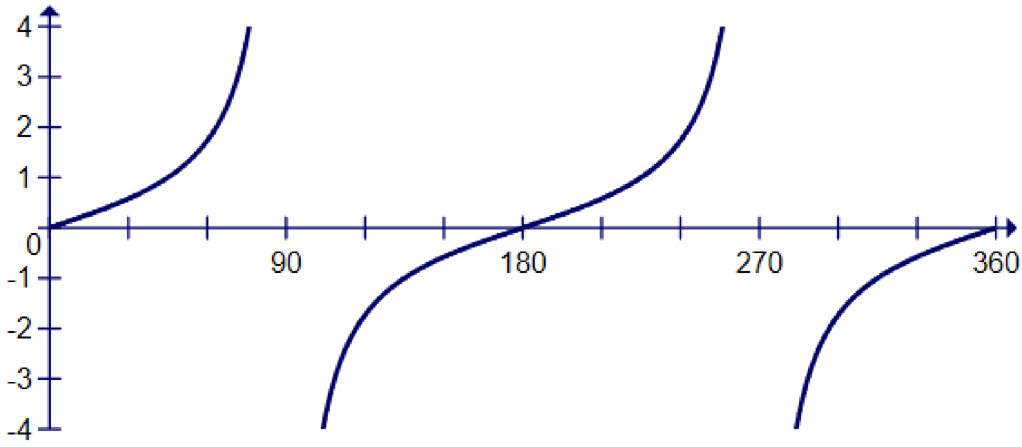
Sine curve



Cosine Curve



Tangent graph



Solving Trigonometric Equations

Example 1 Solve the equation $\sin x° = -0.5$ for $0 \leq x < 360$.

Solution The calculator gives $\sin^{-1}(0.5) = -30$.

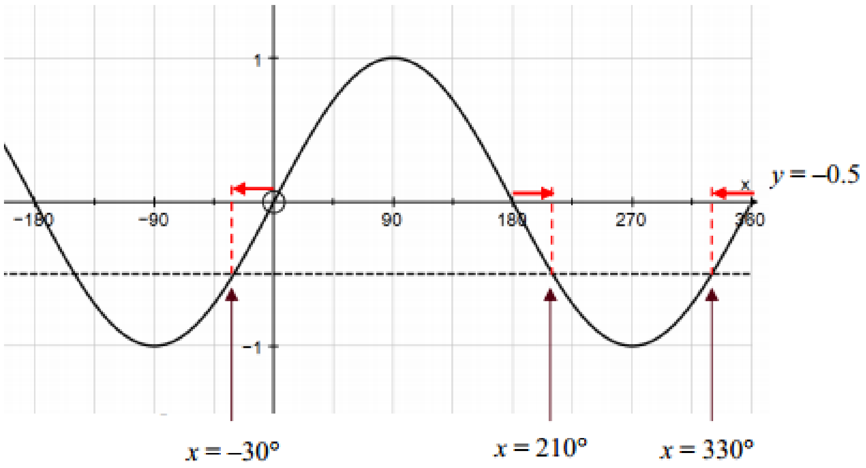
This is usually called the *principal value* of the function $\sin^{-1}$.

To get a second solution you use a graph

Use the graph of $y = \sin x$

By drawing the line $y = -0.5$ on the same set of axes as the graph of the sine curve, points of intersection can be identified in the range

$0 \leq x < 360$.



Exercise F

1 Solve the following equations for $0 \leq x < 360$. Give your answers to the nearest 0.1°.

(a) $\sin x° = 0.9$ (b) $\cos x° = 0.6$ (c) $\tan x° = 2$